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LIMIT CYCLES BIFURCATING FROM THE PERIODIC ANNULUS OF CUBIC HOMOGENEOUS POLYNOMIAL CENTERS

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Abstract. We obtain an explicit polynomial whose simple positive real roots provide the limit cycles which bifurcate from the periodic orbits of any cubic homogeneous polynomial center when it is perturbed inside the class of all polynomial differential systems of degree n.

1. Introduction and statement of main results

One of the main goals in the qualitative theory of real planar differential systems is the determination of their limit cycles. It is well known that perturbing the periodic orbits of a center often produces limit cycles, see for instance [1, 2, 12]. One of the first in studying these perturbations was Pontrjagin [10]. These last years this problem has been studied by many authors see the second part of the book [5] and the hundreds of references quoted there.

Hilbert in 1900 was interested in the maximum number of the limit cycles that a polynomial differential system of a given degree can have. This problem is the well-known 16-th Hilbert problem, which together with the Riemann conjecture are the two problems of the famous list of 23 problems of Hilbert which remain open. See for more details [7, 13].

There exist several methods to study the number of limit cycles that bifurcate from the periodic annulus of a center, such as the *Poincaré return map*, the *Poincaré-Melnikov integrals*, the *Abelian integrals*, the *inverse integrating factor*, and the *averaging theory*. In the plane all of them are essentially equivalent.

There are few works trying to study this problem for homogeneous cubic polynomial differential systems. Our main objetive will be to solve this problem for the cubic homogeneous polynomial differential systems.

In [6] the authors classified all the cubic homogeneous polynomial differential systems. In [8] the authors proved that any real planar cubic homogeneous polynomial differential system having a center can be written as

$$\begin{aligned} \dot{x} &= ax^3 + (b - 3\alpha\mu)x^2y - axy^2 - \alpha y^3 = P(x, y), \\ \dot{y} &= \alpha x^3 + ax^2y + (b + 3\alpha\mu)xy^2 - ay^3 = Q(x, y), \end{aligned} \tag{1.1}$$

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