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LIMIT CYCLES BIFURCATING FROM THE PERIODIC ORBITS OF THE WEIGHT-HOMOGENEOUS POLYNOMIAL CENTERS OF WEIGHT-DEGREE 3

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Abstract. In this article we obtain two explicit polynomials, whose simple positive real roots provide the limit cycles which bifurcate from the periodic orbits of a family of polynomial differential centers of order 5, when this family is perturbed inside the class of all polynomial differential systems of order 5, whose average function of first order is not zero. Then the maximum number of limit cycles that bifurcate from these periodic orbits is 6 and it is reached.

This family of of centers completes the study of the limit cycles which can bifurcate from periodic orbits of all centers of the weight-homogeneous polynomial differential systems of weight-degree 3 when perturbed in the class of all polynomial differential systems having the same degree and whose average function of first order is not zero.

1. Introduction and statement of the main results

One of the main goals in the qualitative theory of real planar polynomial differential systems is the determination of their limit cycles. Studying the number of limit cycles of a polynomial differential system is strongly motivated by Hilbert's 16-th problem (1900). For more details see [8] and [14].

Many authors have studied the number of limit cycles which may bifurcate from the periodic orbits of a center of a polynomial differential system when it is perturbed up to first order in the parameter of the perturbation. This problem is known as the *weak Hilbert's problem*. See for example [1, 3].

Among the many tools for studying the maximum number of limit cycles that may bifurcate from the periodic annulus of a center we have the Poincaré return map, the Poincaré-Melnikov integrals, the Abelian integrals, and the averaging theory. The last three methods are equivalent at first order, see for instance [7]. For studies on the weak Hilbert's problem see, for example, the second part of [6] and the hundreds of references quoted therein.

Here we consider the *polynomial differential systems*

$$\begin{aligned} \dot{x} &= P(x,y),\\ \dot{y} &= Q(x,y), \end{aligned} \tag{1.1}$$

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