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On the Number of Singular Points of the Radial Projection of Polynomial Gradient Vector Fields of $\mathbb{R}^3$ on the Sphere $\mathbb{S}^2$

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Abstract

If $r = (x, y, z)$ and H is a real homogeneous polynomial of degree $m \geq 1$ in the variables (x, y, z), then we prove that the maximum number of pairs of diametrally opposite singular points of the radial projection of the polynomial gradient vector field $\nabla H(r)$ in $\mathbb{R}^3$ over the 2–dimensional sphere $\mathbb{S}^2$ is $(m-1)^2 + (m-1) + 1$, of course when this number is finite. This answers a question of C. Chicone, see [**?**] page 51.

Key words: Polynomial gradient vector fields, singular points.

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1 Introduction

Let H be a real homogeneous polynomial of degree $m \geq 1$ in the variables (x, y, z). In what follows and as it is usual H_u with $u \in \{x, y, z\}$ will denote the partial derivative of H with respect to u.

Let $r = (x, y, z)$. We consider the radial projection of the homogeneous polynomial gradient vector field

$$\nabla_r H(r) = (H_x, H_y, H_z)$$

of degree $m-1$ in $\mathbb{R}^3$ over the 2–dimensional sphere $\mathbb{S}^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2+y^2+z^2 = 1\}$; i.e. we consider the polynomial vector field $\mathcal{X}$ of degree $m+1$ on $\mathbb{S}^2$ defined by

$$\mathcal{X}(r) = \nabla_r H(r) - (r \cdot \nabla_r H(r))\, r \tag{1}$$