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The geometry of the real planar polynomial differential systems having their orbits embedded in conics

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We classify and provide the global phase portraits in the Poincaré disc of all real planar polynomial differential systems having their orbits embedded in conics. This is achieved via the real affine classification of the pencils of conics, and each type corresponds to an equisingularity type of pencil. All such polynomial vector fields have degree less than or equal to 3. Also, when the degree is 3, infinity is filled with singular points.

Keywords: polynomial vector field; phase portraits; rational function; pencil of conics

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1. Introduction and statement of the main results

We consider *real planar polynomial differential systems* or simply *polynomial systems*, i.e. differential systems of the form

$$\frac{dx}{dt} = P(x, y), \quad \frac{dy}{dt} = Q(x, y), \tag{1}$$

where $P(x, y)$ and $Q(x, y)$ are real polynomials in x and y. The *degree* m of (1) is the maximum of the degrees of the polynomials P and Q, i.e. $m = \max\{\deg P, \deg Q\}$. We say that the polynomial differential system (1) is *non-degenerate* if P and Q are coprime, otherwise we say that it is *degenerate*.

Real planar polynomial differential systems appear in many areas of applied mathematics. Only on quadratic polynomial differential systems more than one thousand articles have been written but the understanding of the dynamics of these systems is far from complete. In this article, we study the global dynamics of the polynomial differential systems having all their orbits embedded in conics. To be more precise we say that system (1) has the orbit γ *embedded in a conic* if there exists a polynomial of degree two $F(x, y) \in \mathbb{R}[x, y]$ such that $\gamma \subset \{F(x, y) = 0\}$.

Although real conics are very simple curves and there are only nine different types of them up to an affine transformation, the differential polynomial systems having their

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