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Darboux integrability of the Lü system

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ABSTRACT

We characterize all the values of the parameters of the Lü system, for which it admits a Darboux first integral.

1. Introduction and statement of the main results

The following real differential system

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= cy - xz, \\ \dot{z} &= -bz + xy, \end{aligned} \tag{1}$$

where $a, b, c \in \mathbb{R}$ are parameters is known as the *Lü system* [1]. It connects the Lorenz system [2,3] with the Chen system [4] and represents a transition from one to the another (see [1] for details).

A number of facts related to the *local* analysis of system (1) are known. For results on Hopf bifurcation see [5]; degenerate Hopf bifurcation have been considered in [6,7]; see also [8] for center conditions on the local center manifold. *Global* dynamics of the Lü system have also been recently considered in [2,9], where the authors studied the existence of invariant algebraic surfaces and determined the dynamics on it, including the infinity.

In this work we further consider the global dynamics of system (1) by studying the integrability of system (1). To be more precise, we characterize all the values of the parameters $a, b, c \in \mathbb{R}$, for which the Lü system admits a *Darboux first integral* (see Theorem 3), i.e. a function of the form (2). In particular we identify, the parameter values of the system for which it admits a *rational* first integral (see Theorem 2). Now, we proceed to present more detailed statements of our main results.

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