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Bifurcation of limit cycles from a two-dimensional center inside $\mathbb{R}^n$

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ABSTRACT

We study the bifurcation of limit cycles from the periodic orbits of a linear differential system in $\mathbb{R}^n$ perturbed inside a class of piecewise linear differential systems, which appears in a natural way in control theory. Our main result shows that at most one limit cycle can bifurcate up to first-order expansion of the displacement function with respect to the small parameter. This upper bound is reached. For proving this result we use the averaging theory in a form where the differentiability of the system is not needed.

1. Introduction

Piecewise linear differential systems appear in a natural way in control theory; see for instance [1–7]. These systems can present complicated dynamical phenomena such as those exhibited by general nonlinear differential systems. Among the main ingredients in the qualitative description of the dynamical behavior of a differential system are the number and the distribution of the limit cycles.

The goal of this paper is to study, in $\mathbb{R}^n$ for all $n \geq 2$, the existence of limit cycles of the control system of the form

$$\dot{x} = A_0 x + \varepsilon F(x), \tag{1}$$

for $|\varepsilon| \neq 0$ a sufficiently small real parameter, where

$$A_0 = \begin{pmatrix} 0 & -1 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix},$$

and $F : \mathbb{R}^n \to \mathbb{R}^n$ is given by

$$F(x) = Ax + \varphi(k^{\mathrm{T}} x) b,$$

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