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On the limit cycles for a class of fourth-order differential equations

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Abstract
We provide sufficient conditions for the existence of periodic solutions of the fourth-order differential equation

$$\ddddot{x} + (1+p^2)\ddot{x} + p^2 x = \varepsilon F(x, \dot{x}, \ddot{x}, \dddot{x}),$$

where p is a rational number different from 0, ε is small and F is a nonlinear function.

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1. Introduction and statement of the main results

The objective of this paper is to study the periodic solutions of the fourth-order differential equation

$$\ddddot{x} + (1+p^2)\ddot{x} + p^2 x = \varepsilon F(x, \dot{x}, \ddot{x}, \dddot{x}), \tag{1}$$

where p is a rational number different from 0, ε is a small real parameter and F is a nonlinear function. The dot denotes derivative with respect to an independent variable t.

Equations (1) appear in many places. For instance, these equations are contained in the differential equations (2) studied by Champneys [8], who analyzes these equations looking mainly for homoclinic orbits.

When $F(u, \dot{u}, \ddot{u}, \dddot{u}) = \pm u^2$, equation (1) can come from the description of the traveling-wave solutions of the Korteweg–de Vries equation with an additional fifth-order dispersive term. Extended fifth-order Korteweg–de Vries equations have been considered in [6, 9, 10, 16, 17, 23]. These equations have been used to describe chains of coupled nonlinear oscillators [25] and most notably gravity-capillary shallow water waves [4, 14, 30]. For other derivations of (1), see [12, 13].

Another nonlinearity is $F(u, \dot{u}, \ddot{u}, \dddot{u}) = \pm u^3$; then equation (1) is called the *extended Fischer–Kolmogorov* equation or the *Swift–Hohenberg* equation, see [5, 15] and in other places, for instance [26] and [2, 7].

In all these previously quoted references, and for the corresponding differential equations (1) there studied, there are no proofs showing the existence of periodic orbits.