*Electronic Journal of Differential Equations*, Vol. 2012 (2012), No. 22, pp. 1–17.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

# LIMIT CYCLES FOR FOURTH-ORDER AUTONOMOUS DIFFERENTIAL EQUATIONS

JAUME LLIBRE, AMAR MAKHLOUF

Abstract. We provide sufficient conditions for the existence of periodic solutions of the fourth-order differential equation

$$\ddddot{x} - (\lambda+\mu)\dddot{x} + (1+\lambda\mu)\ddot{x} - (\lambda+\mu)\dot{x} + \lambda\mu x = \varepsilon F(x, \dot{x}, \ddot{x}, \dddot{x}),$$

where $\lambda$, $\mu$ and $\varepsilon$ are real parameters, $\varepsilon$ is small and $F$ is a nonlinear function.

## 1. Introduction and statement of the main results

The objective of this paper is to study the periodic solutions of the fourth-order differential equation

$$\ddddot{x} - (\lambda+\mu)\dddot{x} + (1+\lambda\mu)\ddot{x} - (\lambda+\mu)\dot{x} + \lambda\mu x = \varepsilon F(x, \dot{x}, \ddot{x}, \dddot{x}), \tag{1.1}$$

where $\lambda$, $\mu$ and $\varepsilon$ are real parameters, $\varepsilon$ is small and $F$ is a nonlinear function. The dot denotes derivative with respect to an independent variable $t$.

There are many papers studying the periodic orbits of fourth–order differential equations, see for instance in [3, 4, 5, 6, 7, 11, 12, 13, 14, 15]. But our main tool for studying the periodic orbits of equation (1.1) is completely different to the tools of the mentioned papers, and consequently the results obtained are distinct and new. We shall use the averaging theory, more precisely Theorem 2.1. Many of the quoted papers dealing with the periodic orbits of four-order differential equations use Schauder's or Leray-Schauder's fixed point theorem, or the nonlocal reduction method, or variational methods.

In general to obtain analytically periodic solutions of a differential system is a very difficult task, usually impossible. Here with the averaging theory this difficult problem for the differential equations (1.1) is reduced to find the zeros of a nonlinear function. We must say that the averaging theory for finding periodic solutions in general does not provide all the periodic solutions of the system. For more information about the averaging theory see section 2 and the references quoted there.

Llibre, Makhlouf and Sellami [8] studied equation (1.1) with the nonlinear function $F(x, \dot{x}, \ddot{x}, \dddot{x}, t)$ which depends explicitly on the independent variable $t$. Here we study the autonomous case using a different approach.

2000 *Mathematics Subject Classification.* 37G15, 37C80, 37C30.
*Key words and phrases.* Periodic orbit; fourth-order differential equation; averaging theory.

Submitted October 12, 2011. Published February 7, 2012.