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Periodic orbits of a non-autonomous quadratic differential system obtained from third-order differential equations

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We provide sufficient conditions for the existence of periodic solutions of the third-order differential equations

$$u''' + (a_1u + a_0)u'' + (b_1u + b_0)u' + c_2u^2 + a_0b_0u = \varepsilon^2 F(t, u, u', u'', \varepsilon),$$

where a_0, a_1, b_0, b_1, c_2 are real arbitrary parameters with $b_0 > 0$, ε is a small parameter and F is a $\frac{2\pi}{\sqrt{b_0}}$-periodic nonlinear function in the variable t. The prime denotes derivative with respect to the independent variable t. Moreover we provide an application.

Keywords: periodic orbit; third-order differential equation; quadratic system; averaging theory

AMS Subject Classifications: 37G15; 37C80; 37C30

1. Introduction and statement of the main results

Quadratic differential systems in $\mathbb{R}^3$ are some of the simplest systems after linear systems and have been extensively studied in the past few years. There are many examples of such systems, see for instance the Lorenz system [1], the Chen system [2], the Liu system [3], the Rössler system [4], the Rikitake system [5] and the Lu system [6], among several others.

One of the most interesting problems related to quadratic differential systems is the study of their limit cycles, i.e. their isolated periodic orbits of the system. It is known that every quadratic differential system in $\mathbb{R}^2$ has finitely many limit cycles, see for instance [7–9]. For quadratic systems in $\mathbb{R}^n$ with $n > 2$ the scenario is very different. There are quadratic systems for $n > 2$ with infinitely many limit cycles, see for instance [10].

In [11], the authors study the nonlinear dynamics including the periodic orbits of a quadratic differential system in $\mathbb{R}^3$ which comes from a third-order differential equation. More precisely, they analyse the third-order differential equations

$$u''' + (a_1u + a_0)u'' + (b_1u + b_0)u' + c_2u^2 + c_1u + c_0 = 0.$$

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