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Periodic orbits of the fourth-order non-autonomous differential equation $u'''' + qu'' + pu = \varepsilon F(t, u, u', u'', u''')$

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ABSTRACT

We provide sufficient conditions for the existence of periodic solutions of the fourth-order differential equation

$$u'''' + qu'' + pu = \varepsilon F(t, u, u', u'', u'''),$$

where q, p and ε are real parameters, ε is small and F is a nonlinear non-autonomous periodic function with respect to t. Moreover we provide some applications.

1. Introduction and statement of the main results

The goal of this paper is to study the periodic solutions of the fourth-order non-autonomous differential equation

$$u'''' + qu'' + pu = \varepsilon F(t, u, u', u'', u'''), \tag{1}$$

where q, p and ε are real parameters, ε is small (i.e. $0 < \varepsilon \ll 1$) and F is a nonlinear function. The prime denotes derivative with respect to an independent variable t.

In general to obtain analytically periodic solutions of a differential system is a very difficult task, usually impossible. Recently the study of the periodic solutions of second-, third- and fourth-order of non-autonomous differential equations has been considered by several authors, see for instance [13,15,16,20]. Here, using the averaging theory we reduce this difficult problem for the differential equation (1) to find the zeros of a nonlinear system of two or four equations. We notice that the averaging theory for finding periodic solutions in general does not provide all the periodic solutions of the system. For more information and details about the averaging theory see section (2) and the references quoted there.

Eq. (1) appear in many contexts. For instance, Champneys [5] analyzes a class of Eq. (1) looking mainly for homoclinic orbits.

When $F = \pm u^3$, Eq. (1) is called the *Extended Fischer–Kolmogorov* equation or the *Swift–Hohenberg* equation see [3,7], and in other places, see for instance the book [14,1,4].

Some results on the periodic orbits for extended Fisher–Kolmogorov and Swift–Hohenberg equations of the form

$$u'''' + qu'' + \alpha(t)u = f(t, u, u', u''), \tag{2}$$

with α and f functions, has been studied in [4] and in the references quoted there.

The differential equation (1) when F does not depend on t with $p < 0$ has been studied in [10] taking $p = -1$ after a rescaling, and with $p > 0$ it has been studied in [8].

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