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Periodic orbits of the spatial anisotropic Manev problem

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We study the periodic orbits of the spatial anisotropic Manev problem, which depend on three parameters. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4771902]

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

The objective of this paper is to study the periodic orbits of the *spatial anisotropic Manev problem* given by the Hamiltonian

$$\mathrm{H} = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) - \frac{1}{\sqrt{\mu(x^2+y^2)+z^2}} - \frac{\varepsilon\beta}{\mu(x^2+y^2)+z^2}, \tag{1}$$

where μ is near 1, $\beta \neq 0$ is a parameter and ε is small.

The dynamics of the planar anisotropic Manev problem was studied in Refs. 4 and 6–8, including information on its periodic orbits, but as far as we know there are no works on the spatial periodic orbits of the Manev problem.

If $\mu = 1$ and $\beta = 0$, we have the spatial Kepler problem, see for instance the book in Ref. 3.

If $\mu \neq 1$ and $\beta = 0$, we have the spatial anisotropic Manev problem, which originally comes from the quantum mechanics, see, for instance, Refs. 1, 2, and 10.

If $\mu = 1$ and $\beta \neq 0$ then we have the spatial Manev problem. One of the advantages of the Manev problem over the Keplerian is that it explains the perihelion advance of the inner planets with the same accuracy as relativity, see Refs. 5, 9, 11–15, and 17–19.

Note that the Hamiltonian (1) is symmetric with respect to the z-axis. Then it is easy to check that the third component $K = xp_y - yp_x$ of the angular momentum is a first integral of the Hamiltonian system with Hamiltonian (1). We shall use this integral K to simplify the analysis of the given axially symmetric perturbed system.

Since μ is near 1 and ε is small, we take $\mu = 1 - \varepsilon$, and doing Taylor series in ε at $\varepsilon = 0$ of the Hamiltonian (1), we obtain

$$\mathrm{H} = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) - \frac{1}{\sqrt{x^2+y^2+z^2}} - \varepsilon\frac{x^2+y^2+2\sqrt{x^2+y^2+z^2}\,\beta}{2(x^2+y^2+z^2)^{3/2}} + O(\varepsilon^2). \tag{2}$$

In the following, we shall use the Delaunay variables for studying easily the periodic orbits of the Hamiltonian system associated to the Hamiltonian (2), see Refs. 3 and 16 or Sec. II for more details on the Delaunay variables. Thus, in Delaunay variables, the Hamiltonian (2) has the form

$$\mathrm{H} = -\frac{1}{2L^2} + \varepsilon P(l, g, k, L, G, K) + O(\varepsilon^2), \tag{3}$$

where $P(l, g, k, L, G, K)$ is equal to

$$\frac{-(G+L)((G^2+K^2)L + 4G\beta) + (G^2-K^2)(G-L)L\cos(2g)}{4G^2L^3(G+L)}, \tag{4}$$

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