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ORIGINAL ARTICLE

Periodic orbits of the generalized Friedmann–Robertson–Walker Hamiltonian systems

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Abstract The averaging theory of first order is applied to study a generalization of the Friedmann–Robertson–Walker Hamiltonian systems with three parameters. We provide sufficient conditions on the three parameters of the generalized system to guarantee the existence of continuous families of periodic orbits parameterized by the energy, and these families are given up to first order in a small parameter.

Keywords Periodic orbit · Averaging theory · Friedmann–Robertson–Walker Hamiltonian system

1 Introduction

The dynamics of the universe is an area of the astrophysics where the application of modern results coming from dynamical systems has been revealed very fruitful, specially in galactic dynamics see for instance the articles (Belmonte et al. 2007; Merritt and Valluri 1996; Papaphilippou and Laskar 1996, 1998; Zhao et al. 1999) and the references quoted there.

Calzeta and Hasi (1993) present analytical and numerical evidence of the existence of chaotic motion for the simplified Friedmann–Robertson–Walker Hamiltonian

$$H = \frac{1}{2}\left(p_Y^2 - p_X^2\right) + \frac{1}{2}\left(Y^2 - X^2\right) + \frac{b}{2}X^2Y^2, \tag{1}$$

which modelates a universe, filled with a conformally coupled but massive real scalar field. Although this model is too simplified to be considered realistic, its simplicity itself makes it an interesting testing ground for the implications of chaos in cosmology, either classical, semiclassical or quantum, see for more details (Calzeta and Hasi 1993). Similar models have been used by Hawking (1985) and Page (1991) to discuss the relationship between the cosmological and thermodynamic arrow of time, in the framework of quantum cosmology.

In problems of galactic dynamics it is usual to consider potentials of the form $V(x^2, y^2)$, i.e. potentials exhibiting a reflection symmetry with respect to both axes, see Pucacco et al. (2008) and the previous articles mentioned on galactic dynamics. For this reason here we generalize the Calzeta–Hasi's model as follows

$$\begin{aligned} H = {} & \frac{1}{2}\left(p_Y^2 - p_X^2\right) + \frac{1}{2}\left(Y^2 - X^2\right) + \frac{a}{4}X^4 \\ & + \frac{b}{2}X^2Y^2 + \frac{c}{4}Y^4. \end{aligned} \tag{2}$$

A general result of the qualitative theory of differential systems states that any orbit or trajectory of a differential system is homeomorphic either to a point, or to a circle, or to a straight line. The orbits homeomorphic to a point are the equilibrium points, and the ones homeomorphic to circles are the periodic orbits. It is well known that these two types of orbits play a relevant role in the dynamics of a differential system, and in general they are easier to study than the orbits homeomorphic to straight lines which sometimes can exhibit a very complicate dynamics. In short, the first analysis for understanding the dynamics of a differential system is to start studying its equilibrium points, its periodic solutions and their kind of stability.

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