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Periodic solutions of second order Hamiltonian systems

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We provide sufficient conditions for the existence of periodic solutions of the second order Hamiltonian system

$$-x'' - \lambda x = \varepsilon V'_x(t, x),$$

where ε is a small parameter, $x \in \mathbb{R}$ and $V(t, x)$ is 2π-periodic in t. Moreover we provide two applications.

Keywords: periodic solution; second order Hamiltonian system; averaging theory

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1. Introduction and statement of the main results

The existence of periodic solutions of the second order Hamiltonian system

$$-x'' - \lambda x = V'_x(t, x),$$

where $x \in \mathbb{R}^m$ and V is 2π-periodic in t, has been extensively studied in the last three decades by the variational and others methods, see [1–23] and the references therein. Here, using the averaging theory, we study the periodic solutions of the second order Hamiltonian system

$$-x'' - \lambda x = \varepsilon V'_x(t, x), \tag{1}$$

where ε is a small parameter, $x \in \mathbb{R}$ and V(t,x) is 2π-periodic in t.

To obtain analytically periodic solutions is in general a very difficult work, usually impossible. The averaging theory reduces this difficult problem for the differential Equation (1) to find the zeros of a nonlinear function. It is known that in general the averaging theory for finding periodic solutions does not provide all the periodic solutions of the system. For more information about the averaging theory, see Section 2 and the references quoted there.

We define the functions

$$f_1(x_0, y_0) = \frac{1}{2\pi r}\int_0^{2\pi s} \sin\left(\frac{rt}{s}\right) V'_x(t, a(t))dt,$$
$$f_2(x_0, y_0) = -\frac{1}{2\pi s}\int_0^{2\pi s} \cos\left(\frac{rt}{s}\right) V'_x(t, a(t))dt,$$

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