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Limit Cycles of a Class of Generalized Liénard Polynomial Equations

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Abstract We prove that the generalized Liénard polynomial differential system

$$\dot{x} = y^{2p-1}, \qquad \dot{y} = -x^{2q-1} - \varepsilon f(x)\, y^{2n-1}, \tag{1}$$

where p, q, and n are positive integers; ε is a small parameter; and $f(x)$ is a polynomial of degree m which can have $[m/2]$ limit cycles, where $[x]$ is the integer part function of x.

Keywords Limit cycles · Polynomial differential systems · Liénard systems · Averaging theory · $(p \cdot q)$–Trigonometric functions

Mathematics Subject Classifications 2010 34C05 · 34C25

1 Introduction and Statement of the Main Results

In 1900, Hilbert [2] in the second part of his 16–th problem, proposed to find a uniform upper bound for the number of limit cycles of all polynomial differential systems of a given degree, and also to study their distribution or configuration in the plane. The 16–th problem, except the one related with the Riemann hypothesis, seems to be the most elusive of Hilbert's problems. It has been one of the main problems in the qualitative theory of planar differential equations during the XX century. Until now, it has not proved the existence of such a uniform upper bound. This problem remains open even for the polynomial differential systems of degree 2. However, it is not difficult to see that any finite configuration of limit cycles is realizable for some polynomial differential system (see for details [6]).

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