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PERIODIC SOLUTIONS OF SOME CLASSES OF CONTINUOUS SECOND–ORDER DIFFERENTIAL EQUATIONS

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Abstract. We study the periodic solutions of the second–order differential equations of the form $\ddot{x} \pm x^n = \mu f(t)$, or $\ddot{x} \pm |x|^n = \mu f(t)$, where $n = 4, 5, \ldots$, $f(t)$ is a continuous T–periodic function such that $\int_0^T f(t)dt \neq 0$, and μ is a positive small parameter. Note that the differential equations $\ddot{x} \pm x^n = \mu f(t)$ are only continuous in t and smooth in x, and that the differential equations $\ddot{x} \pm |x|^n = \mu f(t)$ are only continuous in t and locally–Lipschitz in x.

1. **Introduction.**

The periodic solutions of the second–order differential equations

$$\ddot{x} + x^3 = f(t), \tag{1}$$

where $f(t)$ is a T–periodic function have been studied by several authors. Thus, Morris [9] proves that if $f(t)$ is C^1 and its average is zero (i.e. $\int_0^T f(t)dt = 0$), then the differential equation (1) has periodic solutions of period kT for all positive integer k. Ding and Zanolin [6] proved the same result without the assumption that the average of $f(t)$ be zero. Almost there is no results on the stability of these periodic solutions, but Ortega [10] proved that the differential equation (1) has finitely many stable periodic solutions of a fixed period.

On the other hand other authors have studied more general problems as the following one: when an equilibrium or a limit cycle of an autonomous differential system can be continued as a periodic solution when the autonomous system is periodically perturbed. This question of persistence is very classical, but for dimension two and for an equilibrium Buică and Ortega [3] found a complete characterization of the persistence of a such periodic solution. These authors use more general results on the persistence of periodic solutions of autonomous systems under periodic

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