Differ Equ Dyn Syst (January–July 2019) 27(1–3):75–82
https://doi.org/10.1007/s12591-017-0375-5

ORIGINAL RESEARCH

Zero-Hopf Periodic Orbit of a Quadratic System of Differential Equations Obtained from a Third-Order Differential Equation

Jaume Llibre[1] · **Ammar Makhlouf**[2]

Published online: 27 June 2017

Abstract We study the zero-Hopf bifurcation of the third-order differential equations

$$x''' + (a_1 x + a_0)x'' + (b_1 x + b_0)x' + x^2 = 0,$$

where a_0, a_1, b_0 and b_1 are real parameters. The prime denotes derivative with respect to an independent variable t. We also provide an estimate of the zero-Hopf periodic solution and their kind of stability. The Hopf bifurcations of these differential systems were studied in [5], here we complete these studies adding their zero-Hopf bifurcations.

Keywords Periodic orbit · Third-order differential equation · Quadratic system · Averaging theory

Mathematics Subject Classification 37G15 · 37C80 · 37C30

Introduction and Statement of the Main Result

Quadratic systems of differential equations in $\mathbb{R}^3$ are some of the simplest systems after the linear ones and have been extensively studied in the last years. There are many examples of such systems, see for instance Lorenz system [19], the Chen system [4], the Liu system [14], the Rössler system [21], the Rikitake system [20], the Lu system [13], among several others.

One of the most interesting problems related with quadratic systems of differential equations is the study of their limit cycles, i.e. of their isolated periodic orbits in the set

✉ Jaume Llibre
jllibre@mat.uab.cat

Ammar Makhlouf
makhloufamar@yahoo.fr

[1] Departament de Matematiques, Universitat Autònoma de Barcelona, Bellaterra, 08193 Barcelona, Catalonia, Spain

[2] Department of Mathematics, University of Annaba, Elhadjar 23, Annaba, Algeria