International Journal of Bifurcation and Chaos, Vol. 30, No. 12 (2020) 2050170 (6 pages)

DOI: 10.1142/S0218127420501709

Zero-Hopf Periodic Orbits for a Rössler Differential System

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Received August 7, 2019; Revised February 20, 2020

We study the zero-Hopf bifurcation of the Rössler differential system

$$\dot{x} = x - xy - z, \quad \dot{y} = x^2 - ay, \quad \dot{z} = b(cx - z),$$

where the dot denotes the derivative with respect to the independent variable t and a, b, c are real parameters.

Keywords: Periodic orbit; Rössler differential system; averaging theory.

1. Introduction and Statement of the Main Result

Rössler, using the geometry of the three-dimensional flows, introduced several differential systems as prototypes of the simplest autonomous differential equations exhibiting chaos. The simplicity of his systems is in the sense of minimal dimension, minimal number of parameters and minimal nonlinearities. Nowadays in MathSciNet appear more than 114 articles with the words 'Rössler system' in the title. In 2006, Letellier *et al.* [2006] did a classification of chaotic attractors in $\mathbb{R}^3$, where they go back to the differential system

$$\dot{x} = x - xy - z, \quad \dot{y} = x^2 - ay, \quad \dot{z} = b(cx - z), \tag{1}$$

already considered by Rössler [Letellier *et al.*, 2006; Rössler, 1976, 1979] in 1977, showing that this system exhibits a pure cut chaos according to their terminology. As usual, the dot denotes derivative with time t. This differential system has three families of zero-Hopf equilibria. Our objective is to study if from these equilibria, some periodic orbits bifurcate.

Our interest in system (1) is motivated due to the fact that frequently the complex dynamics of some chaotic nonlinear systems started in their equilibria. More precisely, Cândido and Llibre in [Cândido & Llibre, 2018] studied the existence of zero-Hopf bifurcations in three-dimensional systems, and numerically they showed that such bifurcations sometimes are the starting bifurcation of a route to the chaotic motion.

Note that system (1) is invariant under the symmetry $(x, y, z) \to (-x, y, -z)$. A *zero-Hopf equilibrium* is an equilibrium point of a three-dimensional autonomous differential system, which has a zero eigenvalue and a pair of purely imaginary eigenvalues.