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An upper bound of the index of an equilibrium point in the plane

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ABSTRACT

We give an upper bound of the index of an isolated equilibrium point of a C^1 vector field in the plane. The vector field is decomposed in gradient and Hamiltonian components. This decomposition is related with the Loewner vector field. Associated to this decomposition we consider the set Π where the gradient and Hamiltonian components are linearly dependent. The number of branches of Π starting at the equilibrium point determines the upper bound of the index.

1. Introduction and the main results

The analysis of the trajectories near a non-hyperbolic isolated equilibrium point of a vector field in the plane is usually studied doing a blow-up of the point, but the method only works for analytic vector fields or more generally Lojasiewicz vector fields, see for instance [5]. A first approach to the dynamic near an equilibrium point is its topological index, for the definition see [5]. Here we give a procedure that can be applied to all C^1 vector fields on the plane. The method is based on the decomposition of the vector field as the difference of a gradient and a Hamiltonian vector field. The relation between the vector fields of this decomposition and the level sets of the Hamiltonian function give information on the structure of the trajectories near the equilibrium point and in particular an upper bound of the index.

Results on the estimation of the index of an equilibrium point are not abundant, see for instance [1–4,10,11] and the references quoted there. In many of these papers, as in our case, other vector fields or functions are used for the estimation: In [1–3] polynomial approximations, and in [10], for analytic systems, the scalar product of the position vector by the vector field.

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