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THE HILBERT NUMBER OF A CLASS OF DIFFERENTIAL EQUATIONS*

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Abstract The notion of Hilbert number from polynomial differential systems in the plane of degree n can be extended to the differential equations of the form

$$\frac{dr}{d\theta} = \frac{a(\theta)}{\displaystyle\sum_{j=0}^{n} a_j(\theta) r^j} \tag{$*$}$$

defined in the region of the cylinder $(\theta, r) \in \mathbb{S}^1 \times \mathbb{R}$ where the denominator of $(*)$ does not vanish. Here $a, a_0, a_1, \ldots, a_n$ are analytic 2π–periodic functions, and the *Hilbert number* $\mathbb{H}(n)$ is the supremum of the number of limit cycles that any differential equation $(*)$ on the cylinder of degree n in the variable r can have. We prove that $\mathbb{H}(n) = \infty$ for all $n \geq 1$.

Keywords Periodic orbit, averaging theory, trigonometric polynomial, Hilbert number.

MSC(2000) 34C29, 37C27.

1. Introduction

In the article [6] Lins Neto studied the following problem posed by Charles Pugh.

Problem 1. *Let $a_0, a_1, \ldots, a_n : \mathbb{S}^1 \to \mathbb{R}$ be continuous 2π–periodic functions and consider the differential equation*

$$\frac{dr}{d\theta} = a_0(\theta) + a_1(\theta) r + \ldots + a_n(\theta) r^n, \tag{1.1}$$

on the cylinder $(\theta, r) \in \mathbb{S}^1 \times \mathbb{R}$. Then the problem is to know the number of isolated periodic solutions (i.e. limit cycles) of the differential equation (1.1) *in function of n.*

Problem 1 was motivated by the Hilbert's 16–th problem (see for instance [3–5]), because some polynomial differential systems in the plane can be reduced to equations (1.1) as all the polynomial differential systems of degree 2 (see for instance the proposition of [6]), all polynomial differential systems with the linear center

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