Acta Appl Math
https://doi.org/10.1007/s10440-020-00346-6

Dynamics of a Competitive Lotka–Volterra Systems in $\mathbb{R}^3$

Jaume Llibre[1] · **Y. Paulina Martínez**[2]

Received: 20 May 2019 / Accepted: 23 June 2020

Abstract We describe the dynamics of the 3-dimensional competitive Lotka–Volterra systems

$$\dot{x} = x(a - x - y - z), \quad \dot{y} = y(b - x - y - z), \quad \dot{z} = z(c - x - y - z),$$

providing the phase portraits for all the values of the parameters a, b and c with $0 < a < b < c$ in the positive octant of the Poincaré ball.

Mathematics Subject Classification Primary 34C07 · Secondary 34C08

Keywords Lotka–Volterra systems · Integrability · Phase portraits · Poincaré ball · Competitive Lotka–Volterra systems

1 Introduction and Statement of the Main Results

We say that a polynomial vector field $X = (P(x, y, z), Q(x, y, z), R(x, y, z))$ in $\mathbb{R}^3$ is quadratic if the maximum of the degrees of the polynomials P, Q and R is 2. A Lotka–Volterra system in $\mathbb{R}^3$ is a quadratic polynomial vector field X with x a factor of P, y a factor of Q, and z a factor of R.

The Lotka–Volterra systems, were initially proposed as a model for studying the interactions between species in two dimension, developed independently by Alfred J. Lotka in 1925 [18] and by Vito Volterra in 1926 [25]. Later on in 1936 Kolmogorov [13] extended

✉ Y.P. Martínez
ymartinez@ubiobio.cl

J. Llibre
jllibre@mat.uab.cat

[1] Departament de Matemàtiques, Facultat de Ciències, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

[2] Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Concepción, Chile