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# Dynamics of a family of Lotka–Volterra systems in $\mathbb{R}^3$

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ABSTRACT

We provide the phase portraits of the 3-dimensional Lotka–Volterra systems

$$\dot{x} = x(y + az), \quad \dot{y} = y(x + z), \quad \dot{z} = bz(-ax + y),$$

for all the values of the parameters $a$ and $b$, in the finite region and in the infinity region through the Poincaré compactification. We also study the integrability of the system.



## 1. Introduction and statement of the main results

We say that a polynomial vector field $\mathcal{X} = (P(x, y, z), Q(x, y, z), R(x, y, z))$ in $\mathbb{R}^3$ is quadratic if the maximum of the degrees of the polynomials $P$, $Q$ and $R$ is 2. A quadratic polynomial vector field $\mathcal{X}$ with $x$ a factor of $P$, $y$ a factor of $Q$, and $z$ a factor of $R$ is by definition a Lotka–Volterra system.

The Lotka–Volterra systems, that are quadratic polynomial differential systems, were initially proposed in $\mathbb{R}^2$ independently by Alfred J. Lotka in 1925 [18] and by Vito Volterra in 1926 [25], as a model for studying the interactions between species. Later on Kolmogorov [12] in 1936 extended these systems to arbitrary dimension and arbitrary degree, these kinds of systems are now called Kolmogorov systems.

Many natural phenomena can be modeled by the Lotka–Volterra systems such as the time evolution of conflicting species in biology [20], chemical reactions [9], hydrodynamics [6], economics [24], the coupling of waves in laser physics [13], the evolution of electrons, ions and neutral species in plasma physics [14], etc. After the work of Brenig and Goriely [4,5] the interest in the Lotka–Volterra systems becomes more

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