Periodic orbits, invariant tori, and cylinders of Hamiltonian systems near integrable ones having a return map equal to the identity

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Generically the return map of an integrable Hamiltonian system with two degrees of freedom in a Hamiltonian level foliated by invariant tori is a twist map. If we perturb such integrable Hamiltonian system inside the class of Hamiltonian systems with two degrees of freedom, then the Poincaré–Birkhoff theorem allows to determine which periodic orbits of the integrable can be prolonged to the perturbed one, and the KAM theory provides sufficient conditions in order that some invariant tori persist under sufficiently small perturbations. If some power of this return map is the identity, then in general for these degenerate Hamiltonian systems we cannot study which periodic orbits of the integrable can be prolonged to the perturbed one, or if some invariant tori persist. This paper studies the perturbation of integrable Hamiltonian systems with two degrees of freedom having some power of the return map equal to the identity. We show with two different models a way to study the prolongation of periodic orbits and of invariant tori or cylinders filled with periodic orbits for such kind of Hamiltonian systems. The main tool in this study is the averaging theory. © *2010 American Institute of Physics.* [doi:10.1063/1.3477937]

I. INTRODUCTION AND STATEMENT OF THE RESULTS

One of the main problems in general perturbation theory is to detect how persistent are some given properties. In other words we want to translate some dynamical properties from the unperturbed system to the perturbed one. Frequently the unperturbed system is linear, polynomial, or an integrable Hamiltonian system and the objects to be continued to the perturbed systems are equilibria, periodic orbits, or invariant tori. In case of nonpersistence, bifurcations occur. Roughly speaking our concern in this paper is to perturb a given integrable Hamiltonian system of two degrees of freedom inside a subclass of Hamiltonian systems with two degrees of freedom, and analyze the continuation of periodic orbits and invariant tori. The precise Hamiltonian systems which will be studied are described below.

It is well known that generically the return map of an integrable Hamiltonian system with two degrees of freedom is twist maps, and that the Poincaré–Birkhoff theorem allows to study which periodic orbits persist when we perturb it inside the class of Hamiltonian systems with two degrees of freedom. On the other hand the KAM theory determines which invariant tori filled with quasiperiodic orbits can be prolonged from the integrable Hamiltonian system to the perturbed one. For more details and definitions about all these notions see, for instance, Refs. 1, 2, and 7.

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