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ON THE BIRTH OF MINIMAL SETS FOR PERTURBED REVERSIBLE VECTOR FIELDS

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Abstract. The results in this paper fit into a program to study the existence of periodic orbits, invariant cylinders and tori filled with periodic orbits in perturbed reversible systems. Here we focus on bifurcations of one-parameter families of periodic orbits for reversible vector fields in $\mathbb{R}^4$. The main used tools are normal forms theory, Lyapunov-Schmidt method and averaging theory.

1. **Introduction.** One of the main objectives of the qualitative theory of differential equations is to study the persistence or bifurcation of minimal invariant sets for a given differential equation under small perturbations. Here the unperturbed systems are reversible vector fields and symmetry breaking bifurcations are considered.

As usual $\mathbb{N}$ denotes the set of positive integers, and $\mathbb{R}$ denotes the set of real numbers. Recall that a differential system

$$\frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}} = f(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^4, \tag{1}$$

is said to be *φ-reversible* if there exists a smooth involution $\varphi : \mathbb{R}^4 \to \mathbb{R}^4$, with $\dim(\mathrm{Fix}(\varphi)) = 2$, such that

$$D\varphi(\mathbf{x})f(\mathbf{x}) = -f(\varphi(\mathbf{x})).$$

Clearly if $\mathbf{x}(t)$ is a solution of system (1), then $\varphi(\mathbf{x}(-t))$ is also a solution of (1). A *solution* $\mathbf{x}(t)$ of (1) is said to be *symmetric* if $\mathbf{x}(t) = \varphi(\mathbf{x}(-t))$. We point out that a solution is symmetric if it intersects $\mathrm{Fix}(\varphi)$ in at least one point. Moreover if $\mathbf{x}(t)$ intersects $\mathrm{Fix}(\varphi)$ in more than one point, then it is a *symmetric periodic orbit*.

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