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Global dynamics of a Lotka–Volterra system in $\mathbb{R}^3$

Jaume Llibre

Departament de Matemàtiques, Facultat de Ciències, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
jllibre@mat.uab.cat

Y. Paulina Martínez

Centre de Recerca Matemática, 08193 Bellaterra, Barcelona, Catalonia, Spain
yohanna.martinez@uab.cat

Claudia Valls

Center for Mathematical Analysis, Geometry and Dynamical Systems, Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisboa, Portugal
cvalls@math.tecnico.ulisboa.pt

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In this work we consider the Lotka–Volterra system in $\mathbb{R}^3$

$$\dot{x} = -x(x-y-z), \quad \dot{y} = -y(-x+y-z), \quad \dot{z} = -z(-x-y+z),$$

introduced recently in [7], and studied also in [8] and [14]. In the first two papers the authors mainly studied the integrability of this differential system, while in the third paper they studied the system as a Hamilton-Poisson system, and also started the analysis of its dynamics. Here we provide the global phase portraits of this 3-dimensional Lotka–Volterra system in the Poincaré ball, that is in $\mathbb{R}^3$ adding its extension to the infinity.

Keywords: Lotka–Volterra system; phase portraits; Poincaré ball.

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1. Introduction and statement of the main results

The Lotka–Volterra systems, developed independently by Alfred J. Lotka in 1925 [9] and Vito Volterra in 1926 [15], were initially proposed as models for studying the interactions in two dimensions between species. Kolmogorov [5] in 1936 extended these systems to arbitrary dimension and degree, which are now called Kolmogorov systems.

The Lotka–Volterra systems have been applied to model different natural phenomena such as the time evolution of conflicting species in biology (which began with the work of May [10]), the evolution of competition between three species (studied by May and Leonard [11]), the evolution of electrons, ions and neutral species in plasma physics [6], chemical reactions [4], hydrodynamics [1], economics [13], etc.