LINEAR TYPE CENTERS OF POLYNOMIAL HAMILTONIAN SYSTEMS WITH NONLINEARITIES OF DEGREE 4 SYMMETRIC WITH RESPECT TO THE Y-AXIS

JAUME LLIBRE, PAULINA MARTÍNEZ, AND CLAUDIO VIDAL

Abstract. We provide normal forms and the phase portraits in the Poincaré disk for all the linear type centers of polynomial Hamiltonian systems with nonlinearities of degree 4 symmetric with respect to the y-axis.

1. Introduction and statement of the main result

In this work we deal with polynomial differential systems in $\mathbb{R}^2$ of the form

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where the dot denotes derivative with respect to an independent real variable t, usually called the *time*. Assume the origin O is an equilibrium point of system (1).

When all the orbits of system (1) in a punctured neighborhood of the equilibrium point O are periodic, we say that the origin is a *center*. The study of the centers started with Poincaré [17] and Dulac [8], and in the present days many questions about them remain open.

If a polynomial system (1) has a center at the origin, then after a linear change of variables and a scaling of the time variable, it can be written in one of the following three forms:

$$\dot{x} = -y + P_2(x, y), \quad \dot{y} = x + Q_2(x, y),$$

called a *linear type center*,

$$\dot{x} = y + P_2(x, y), \quad \dot{y} = Q_2(x, y),$$

called a *nilpotent center*,

$$\dot{x} = P_2(x, y), \quad \dot{y} = Q_2(x, y),$$

called a *degenerate center*, where $P_2(x, y)$ and $Q_2(x, y)$ are polynomials without constant and linear terms.

The classification of the centers of quadratic differential systems (which all of them are linear type centers) started with the works of Dulac [8], Kapteyn [11, 12] and Bautin [3], and the characterization of their phase portraits in the Poincaré disk was due to Vulpe [20], see also Schlomiuk [19]. There are many partial results for the centers of polynomial differential systems of degree larger than 2. We must mention that Malkin [13], and Vulpe and Sibirsky [21] characterized the linear type centers of the polynomial differential systems with linear and homogeneous nonlinearities of degree 3. For polynomial differential systems of the form linear

2010 *Mathematics Subject Classification.* Primary 34C07, Secondary 34C08.

Key words and phrases. Hamiltonian systems, linear type centers, quartic polynomial, polynomial vector fields, phase portraits.