PHASE PORTRAITS OF LINEAR TYPE CENTERS OF POLYNOMIAL HAMILTONIAN SYSTEMS WITH HAMILTONIAN FUNCTION OF DEGREE 5 OF THE FORM $H = H_1(x) + H_2(y)$

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Abstract. We study the phase portraits on the Poincaré disc for all the linear type centers of polynomial Hamiltonian systems of degree 5 with Hamiltonian function $H(x,y) = H_1(x) + H_2(y)$, where $H_1(x) = \frac{1}{2}x^2 + \frac{a_3}{3}x^3 + \frac{a_4}{4}x^4 + \frac{a_5}{5}x^5$ and $H_2(y) = \frac{1}{2}y^2 + \frac{b_3}{3}y^3 + \frac{b_4}{4}y^4 + \frac{b_5}{5}y^5$ as function of the six real parameters a_3, a_4, a_5, b_3, b_4 and b_5 with $a_5b_5 \neq 0$.

1. Introduction and statement of the main result

Consider the polynomial differential systems in $\mathbb{R}^2$ of the form

$$\dot{x} = P(x,y), \quad \dot{y} = Q(x,y), \tag{1}$$

the dot denotes derivative with respect to an independent real variable t, usually called the *time*. We assume that the origin $(0,0)$ is an equilibrium of system (1).

We say that the origin is a *center* if there exists a neighborhood U of the origin such that all the orbits of system (1) in $U \setminus \{(0,0)\}$ are periodic. Poincaré [23] and Dulac [11] began the study of this type of equilibria, in the present days many questions on the centers remain open.

If the origin of system (1) is a center, then after introducing a linear change of variables and a scaling of the time, system (1) can be carried to one of three normal forms:

$$\dot{x} = -y + P_2(x,y), \quad \dot{y} = x + Q_2(x,y),$$

called a *linear type center*,

$$\dot{x} = y + P_2(x,y), \quad \dot{y} = Q_2(x,y),$$

called a *nilpotent center*,

$$\dot{x} = P_2(x,y), \quad \dot{y} = Q_2(x,y),$$

called a *degenerate center*, where $P_2(x,y)$ and $Q_2(x,y)$ are polynomials without constant and linear terms.

In this work we deal with a particular polynomial differential systems in $\mathbb{R}^2$ of the form

$$\dot{x} = -y + \tilde{P}_1(y), \quad \dot{y} = x + \tilde{P}_2(x), \tag{2}$$

where $\tilde{P}_1(y)$ and $\tilde{P}_2(x)$ are polynomials without constant and linear terms.

The classification of centers of quadratic differential systems started with the works of Dulac [11], Kapteyn [15, 16] and Bautin [4], and the characterization of their phase portraits in the Poincaré disc was due to Vulpe [25]. There are many partial results for the centers of polynomial differential systems of degree larger than 2. Malkin [18], and Vulpe and Sibirsky [26] characterized the linear type centers of the polynomial differential systems with linear and homogeneous nonlinearities of degree 3. The centers for

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