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On the invariant hyperplanes for *d*-dimensional polynomial vector fields

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Abstract

We deal with polynomial vector fields $\mathcal{X}$ of the form $\sum_{k=1}^{d} P_k(x_1, \ldots, x_d)\partial/\partial x_k$ with $d \geqslant 2$. Let m_k be the degree of P_k. We call $(m_1, \ldots, m_d)$ the degree of $\mathcal{X}$. We provide the best upper bounds for the polynomial vector field $\mathcal{X}$ in the function of its degree $(m_1, \ldots, m_d)$ of (1) the maximal number of invariant hyperplanes, (2) the maximal number of parallel invariant hyperplanes, and (3) the maximal number of invariant hyperplanes that pass through a single point. Moreover, if $m_i = m$, $i = 1, \ldots, d$, we show that these best upper bounds are reached taking into account the multiplicity of the invariant hyperplanes.

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1. Introduction and statement of the results

As usual we denote by $\mathbb{C}[x_1, \ldots, x_d]$ the ring of the polynomials in the variables $x_1, \ldots, x_d$ with coefficients in $\mathbb{C}$. By definition a *polynomial differential system* in $\mathbb{C}^d$ is a system of the form

$$\frac{dx_i}{dt} = P_i(x_1, \ldots, x_d), \qquad i = 1, \ldots, d, \tag{1}$$

where $P_i \in \mathbb{C}[x_1, \ldots, x_d]$. If m_i is the degree of P_i, then we say that $\mathbf{m} = (m_1, \ldots, m_d)$ is the *degree* of the polynomial system. Without loss of generality in the rest of the paper we assume that $m_1 \geqslant \cdots \geqslant m_d$.

We denote by

$$\mathcal{X} = \sum_{i=1}^{d} P_i(x_1, \ldots, x_d)\frac{\partial}{\partial x_i} \tag{2}$$

the polynomial vector field associated with system (1) of degree $\mathbf{m}$.

