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Limit cycles, invariant meridians and parallels for polynomial vector fields on the torus ☆

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Abstract

We study the polynomial vector fields of arbitrary degree in $\mathbb{R}^3$ having the 2-dimensional torus

$$\mathbb{T}^2 = \left\{(x, y, z) \in \mathbb{R}^3\colon \left(x^2 + y^2 - a^2\right)^2 + z^2 = 1\right\} \quad \text{with } a > 1,$$

invariant by their flow. We characterize all the possible configurations of invariant meridians and parallels that these vector fields can exhibit. Furthermore we analyze when these invariant either meridians or parallels can be limit cycles.

MSC: 58F14; 58F22; 34C05

Keywords: Polynomial vector fields; Invariant parallel; Invariant meridian; Limit cycle; Periodic orbit

1. Introduction and statement of the results

Polynomial vector fields or equivalently polynomial differential equations in the plane have been intensively studied since 1900 due to the second part of the 16th Hilbert problem, which mainly states: Provide un upper bound for the maximum number of limit cycles that a given polynomial vector field of degree n can exhibit in function of n; for more details see [6,7,9].

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