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Limit cycles for Singular Perturbation Problems via Inverse Integrating Factor

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ABSTRACT: In this paper singularly perturbed vector fields X_ε defined in $\mathbb{R}^2$ are discussed. The main results use the solutions of the linear partial differential equation $X_\varepsilon V = \text{div}(X_\varepsilon)V$ to give conditions for the existence of limit cycles converging to a singular orbit with respect to the Hausdorff distance.

Key Words: Limit cycles, vector fields, singular perturbation, inverse integrating factor.

Contents

1. Introduction and statement of the main results

The present work fits within the geometric study of singular perturbation problems expressed by one–parameter families of vector fields $X_\varepsilon : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ where

$$X_\varepsilon(x,y) = (f(x,y,\varepsilon), \varepsilon g(x,y,\varepsilon)) \tag{1}$$

with $\varepsilon \geq 0$, $f, g \in C^r$ for $r \geq 1$ or $f, g \in C^\varpi$ for which we want to study the phase portrait, for sufficient small ε, near the set of singular points of X_0 :

$$\Sigma = \{(x,y) \in \mathbb{R}^2 : f(x,y,0) = 0\}.$$

Special emphasis will be given on systems which the solutions of the linear partial differential equation

$$X_\varepsilon V := f\frac{\partial V}{\partial x} + \varepsilon g \frac{\partial V}{\partial y} = \text{div}\,(X_\varepsilon)\,V$$

are known.

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