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On the periodic orbits of perturbed Hooke Hamiltonian systems with three degrees of freedom

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Dedicated to a friend of both authors, Jorge Sotomayor, on the occasion of his 70th birthday

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ABSTRACT

We study periodic orbits of Hamiltonian differential systems with three degrees of freedom using the averaging theory. We have chosen the classical integrable Hamiltonian system with the Hooke potential and we study periodic orbits which bifurcate from the periodic orbits of the integrable system perturbed with a non-autonomous potential.

1. Introduction

In this paper we study the spatial motion of a particle of unitary mass under the action of a central force with Hamiltonian given by

$$H_0(x, y, z, p_x, p_y, p_z) = \frac{1}{2}\left(p_x^2 + p_y^2 + p_z^2\right) + V_0\left(\sqrt{x^2 + y^2 + z^2}\right),$$

perturbed by the Hamiltonian

$$H(x, y, z, p_x, p_y, p_z, t) = H_0(x, y, z, p_x, p_y, p_z) + \varepsilon V(t, x, y, z), \tag{1}$$

where ε is a small parameter and $V(t, x, y, z)$ is a perturbation of the potential eventually depending on the time t.

We consider a central force derived from a potential of the form

$$V_0\left(\sqrt{x^2 + y^2 + z^2}\right) = \pm(x^2 + y^2 + z^2)^{\alpha/2}, \tag{2}$$

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