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LIMIT CYCLES FOR DISCONTINUOUS GENERALIZED LIENARD POLYNOMIAL DIFFERENTIAL EQUATIONS

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Abstract. We divide $\mathbb{R}^2$ into sectors $S_1, \dots, S_l$, with $l > 1$ even, and define a discontinuous differential system such that in each sector, we have a smooth generalized Lienard polynomial differential equation $\ddot{x} + f_i(x)\dot{x} + g_i(x) = 0$, $i = 1, 2$ alternatively, where f_i and g_i are polynomials of degree $n-1$ and m respectively. Then we apply the averaging theory for first-order discontinuous differential systems to show that for any n and m there are non-smooth Lienard polynomial equations having at least $\max\{n, m\}$ limit cycles. Note that this number is independent of the number of sectors.

Roughly speaking this result shows that the non-smooth classical ($m = 1$) Lienard polynomial differential systems can have at least the double number of limit cycles than the smooth ones, and that the non-smooth generalized Lienard polynomial differential systems can have at least one more limit cycle than the smooth ones.

1. Introduction

A large number of problems from mechanics and electrical engineering, theory of automatic control, economy, impact systems among others cannot be described with smooth dynamical systems. This fact has motivated many researchers to the study of qualitative aspects of the phase space of non-smooth dynamical systems.

One of the main problems in the qualitative theory of real planar continuous and discontinuous differential systems is the determination of their limit cycles. The non-existence, existence, uniqueness and other properties of limit cycles have been studied extensively by mathematicians and physicists, and more recently also by chemists, biologists, economists, etc (see for instance the books [2, 5, 24]). This problem restricted to continuous planar polynomial differential equations is the well known 16th Hilbert's problem [10]. Since this Hilbert's problem turned out a strongly difficult one Smale [23] particularized it to Lienard polynomial differential equations in his list of problems for the present century.

The classical Lienard polynomial differential equations

$$\ddot{x} + f(x)\dot{x} + g(x) = 0, \tag{1.1}$$

where $f(x)$ and $g(x) = x$ goes back to [11]. The dot denotes differentiation with respect to the time t. This second-order differential equation (1.1) can be written

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