J. Math. Anal. Appl. 413 (2014) 763–775

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Limit cycles for discontinuous quadratic differential systems with two zones

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ARTICLE INFO

Article history:
Received 26 February 2013
Available online 16 December 2013
Submitted by C.E. Wayne

Keywords:
Limit cycles
Discontinuous quadratic systems
Averaging theory
Isochronous center

ABSTRACT

In this paper we study the maximum number of limit cycles given by the averaging theory of first order for discontinuous differential systems, which can bifurcate from the periodic orbits of the quadratic isochronous centers $\dot{x} = -y + x^2$, $\dot{y} = x + xy$ and $\dot{x} = -y + x^2 - y^2$, $\dot{y} = x + 2xy$ when they are perturbed inside the class of all discontinuous quadratic polynomial differential systems with the straight line of discontinuity $y = 0$. Comparing the obtained results for the discontinuous with the results for the continuous quadratic polynomial differential systems, this work shows that the discontinuous systems have at least 3 more limit cycles surrounding the origin than the continuous ones.

1. Introduction

One of the main problems in the qualitative theory of continuous planar polynomial differential systems is the study of their limit cycles, see for instance [13]. The limit cycles of continuous planar quadratic polynomial differential systems has been studied intensively, see for instance the books [9,19] and the hundreds of references quoted therein.

The classification of the quadratic polynomial differential systems having an isochronous center is due to Loud [18]. He proved that after an affine change of variables and a rescaling of the independent variable any quadratic isochronous center can be written as one of the four systems of Table 1.

Chicone and Jacobs proved in [8] that at most 2 limit cycles bifurcate from the periodic orbits of the isochronous center

$$\dot{x} = -y + x^2, \qquad \dot{y} = x + xy, \tag{1}$$

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0022-247X/$ – see front matter
http://dx.doi.org/10.1016/j.jmaa.2013.12.031