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INVARIANT TORI FULFILLED BY PERIODIC ORBITS FOR FOUR-DIMENSIONAL $\mathcal{C}^2$ DIFFERENTIAL SYSTEMS IN THE PRESENCE OF RESONANCE

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We provide an algorithm for studying invariant tori fulfilled by periodic orbits of a perturbed system which emerge from the set of periodic orbits of an unperturbed linear system in $p : q$ resonance. We illustrate the algorithm with an application.

Keywords: Periodic orbit; center; resonance; invariant tori; averaging theory.

1. Introduction and Statement of the Results

One of the main problems in general perturbation theory is to detect how persistent are some given properties. In other words, we want to translate some dynamical properties from the unperturbed system to the perturbed one. Frequently the unperturbed system is linear and the objects to be continued to the perturbed system are equilibria, periodic orbits or invariant tori. Bifurcations appear when the nonpersistence occurs.

Our goal in this note is to provide an algorithm for studying the invariant tori fulfilled by periodic orbits of the perturbed system which emerge from the set of periodic orbits of the unperturbed system. Usually the persistence of invariant tori is for tori fulfilled by quasiperiodic orbits, as for instance, the KAM tori. But here we study the persistence of invariant tori fulfilled by periodic orbits.

We consider the four-dimensional linear center

$$\dot{\mathbf{x}} = A\mathbf{x}, \tag{1}$$

where

$$A = \begin{pmatrix} 0 & -p & 0 & 0 \\ p & 0 & 0 & 0 \\ 0 & 0 & 0 & -q \\ 0 & 0 & q & 0 \end{pmatrix},$$

where $\mathbf{x} = (x, y, z, w) \in \mathbb{R}^4$, and p and q are coprime positive integers. Clearly all orbits of system (1) are periodic with the exception of its unique singular point located at the origin of coordinates. We say that the periodic orbits of this center are in *resonance* $p : q$.