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Limit cycles of resonant four-dimensional polynomial systems

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We study the bifurcation of limit cycles from four-dimensional centres inside a class of polynomial differential systems. Our results establish an upper bound for the number of limit cycles which can be prolonged in function of the degree of the polynomial perturbation considered, up to first-order expansion of the displacement function with respect to small parameter. The main tool for proving such results is the averaging theory.

Keywords: limit cycle; periodic orbit; averaging theory; resonance

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1. Introduction

The problem of determining the maximum number of limit cycles that a given differential system can have become one of the main topics in the qualitative theory of differential systems.

The second part of Hilbert's 16th problem is, roughly speaking, to find a uniform upper bound for the number of limit cycles that a planar polynomial differential system with a given degree can have. Related to this problem there exists a special interest given in the following question: *How many limit cycles emerge from a perturbation of a planar centre?* This problem has been studied by many researchers and many different results were obtained, see, for example, [1] and the references therein. Our main concern is to bring this problem to a higher dimension.

We consider the following problem: *How many limit cycles emerge from the periodic orbits of a centre in* $\mathbb{R}^4$ *when we perturb it inside a given class of polynomial differential systems?* More precisely, we consider the four-dimensional linear centre:

$$\dot{x} = Ax, \tag{1}$$

where

$$A = \begin{pmatrix} 0 & -p & 0 & 0 \\ p & 0 & 0 & 0 \\ 0 & 0 & 0 & -q \\ 0 & 0 & q & 0 \end{pmatrix},$$

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