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Large amplitude oscillations for a class of symmetric polynomial differential systems in $\mathbb{R}^3$

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ABSTRACT

In this paper we study a class of symmetric polynomial differential systems in $\mathbb{R}^3$, which has a set of parallel invariant straight lines, forming degenerate heteroclinic cycles, which have their two singular endpoints at infinity. The global study near infinity is performed using the Poincaré compactification. We prove that for all $n \in \mathbb{N}$ there is $\varepsilon_n > 0$ such that for $0 < \varepsilon < \varepsilon_n$ the system has at least n large amplitude periodic orbits bifurcating from the heteroclinic loop formed by the two invariant straight lines closest to the x-axis, one contained in the half-space $y > 0$ and the other in $y < 0$.

Key words: infinite heteroclinic loops, periodic orbits, symmetric systems.

1 INTRODUCTION

In this paper we study the following class of symmetric polynomial differential systems in $\mathbb{R}^3$

$$
\begin{aligned}
\dot{x} &= \frac{dx}{dt} = y, \\
\dot{y} &= \frac{dy}{dt} = z, \\
\dot{z} &= \frac{dz}{dt} = p(y) + \varepsilon\, q(x)\, z,
\end{aligned}
\tag{1}
$$

where ε is a small positive real parameter,

$$
p(y) = \sum_{i=0}^{m} a_i y^i \qquad \text{and} \qquad q(x) = \sum_{i=1}^{m/2} b_{2i-1} x^{2i-1}, \tag{2}
$$

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