PERIODIC AND ASYMPTOTIC ORBITS FOR REVERSIBLE SYSTEMS IN $\mathbb{R}^3$ WITHOUT SINGULAR POINTS

JAUME LLIBRE and MARCELO MESSIAS

Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain. E-mail: jllibre@mat.uab.cat

Depto. de Matemática, Estatística e Computação, FCT/UNESP, Cx. P. 467, 19060–900, P.Prudente-SP, Brazil. E-mail: marcelo@fct.unesp.br

We consider a class of reversible polynomial differential systems in $\mathbb{R}^3$ without singular points and having a set of parallel invariant straight lines forming degenerate heteroclinic cycles, which have their two singular endpoints at infinity. The systems considered depend on a small real parameter ε. It is proved that for all $n \in \mathbb{N}$ there is $\varepsilon_n > 0$ such that for $0 < \varepsilon < \varepsilon_n$ the systems have at least n large amplitude periodic orbits bifurcating from one of the heteroclinic cycles. Furthermore, it is proved that, under certain geometric conditions, there exists a set of trajectories tending asymptotically to one of the invariant straight lines as both $t \to \pm\infty$ (which we call asymptotic orbits in the title). These type of solutions was observed and analytically proved to exist for certain equations coming from problems in fluid mechanics. In this work the geometrical mechanism behind their existence is presented. The global analysis near infinity is performed using the Poincaré compactification in $\mathbb{R}^3$.

Keywords: Reversible system; heteroclinic loop; periodic orbits; Poincaré compactification; asymptotic orbits.

1. Introduction and statement of the results

In this note we consider the following class of symmetric polynomial differential systems in $\mathbb{R}^3$

$$\begin{aligned} \dot{x} &= \frac{dx}{dt} = y, \\ \dot{y} &= \frac{dy}{dt} = z, \\ \dot{z} &= \frac{dz}{dt} = p(y) + \varepsilon\, q(x)\, z, \end{aligned} \tag{1}$$