

# Quadratic three-dimensional differential systems having invariant planes with total multiplicity nine

Jaume Llibre<sup>1</sup> · Marcelo Messias<sup>2</sup>  ·  
Alisson C. Reinol<sup>3</sup>

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**Abstract** In this paper we consider all the quadratic polynomial differential systems in  $\mathbb{R}^3$  having exactly nine invariant planes taking into account their multiplicities. This is the maximum number of invariant planes that these kind of systems can have, without taking into account the infinite plane. We prove that there exist thirty possible configurations for these invariant planes, and we study the realization and the existence of first integrals for each one of these configurations. We show that at least twenty three of these configurations are realizable and provide explicit examples for each one of them.

**Keywords** Polynomial differential systems · Invariant planes · First integrals · Extactic polynomial

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✉ Marcelo Messias  
marcelo@fct.unesp.br

Jaume Llibre  
jllibre@mat.uab.cat

Alisson C. Reinol  
alissoncarv@gmail.com

<sup>1</sup> Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra Barcelona, Catalonia, Spain

<sup>2</sup> Departamento de Matemática e Computação, Faculdade de Ciências e Tecnologia, UNESP – Universidade Estadual Paulista, Presidente Prudente, SP, Brazil

<sup>3</sup> Departamento de Matemática, Instituto de Biociências, Letras e Ciências Exatas, UNESP – Universidade Estadual Paulista, São José do Rio Preto, SP, Brazil