J. Phys. A: Math. Theor. **41** (2008) 275210 (21pp) doi:10.1088/1751-8113/41/27/275210

On the global dynamics of the Rabinovich system

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Received 17 March 2008
Published 16 June 2008
Online at stacks.iop.org/JPhysA/41/275210

Abstract

In this paper by using the Poincaré compactification in $\mathbb{R}^3$ we make a global analysis of the Rabinovich system
$\dot{x} = hy - v_1x + yz, \qquad \dot{y} = hx - v_2y - xz, \qquad \dot{z} = -v_3z + xy,$
with $(x, y, z) \in \mathbb{R}^3$ and $(h, v_1, v_2, v_3) \in \mathbb{R}^4$. We give the complete description of its dynamics on the sphere at infinity. For ten sets of the parameter values the system has either first integrals or invariants. For these ten sets we provide the global phase portrait of the Rabinovich system in the Poincaré ball (i.e. in the compactification of $\mathbb{R}^3$ with the sphere $\mathbb{S}^2$ of the infinity). We prove that for convenient values of the parameters the system has two families of singularly degenerate heteroclinic cycles. Then changing slightly the parameters we numerically found a four wings butterfly shaped strange attractor.

(Some figures in this article are in colour only in the electronic version)

1. Introduction and statement of the main results

The Rabinovich system is the four-parameter family of quadratic differential equations given by

$$\dot{x} = hy - v_1x + yz, \qquad \dot{y} = hx - v_2y - xz, \qquad \dot{z} = -v_3z + xy, \tag{1}$$

where the state variables $(x, y, z) \in \mathbb{R}^3$ and the parameters $(h, v_1, v_2, v_3) \in \mathbb{R}^4$. As usual the dots denote a derivative with respect to the time t.

System (1) was first studied in [12] throughout the analysis of a concrete realization in a magnetoactive nonisothermal plasma. From the physical point of view, it is a dynamical system of three resonantly coupled waves, parametrically excited. Numerically we get parameter

