# Global dynamics of stationary solutions of the extended Fisher–Kolmogorov equation

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In this paper we study the fourth order differential equation $\frac{d^4u}{dt^4} + q\frac{d^2u}{dt^2} + u^3 - u = 0$, which arises from the study of stationary solutions of the Extended Fisher–Kolmogorov equation. Denoting $x = u$, $y = \frac{du}{dt}$, $z = \frac{d^2u}{dt^2}$, $v = \frac{d^3u}{dt^3}$ this equation becomes equivalent to the polynomial system $\dot{x} = y$, $\dot{y} = z$, $\dot{z} = v$, $\dot{v} = x - qz - x^3$ with $(x, y, z, v) \in \mathbb{R}^4$ and $q \in \mathbb{R}$. As usual, the dot denotes the derivative with respect to the time $t$. Since the system has a first integral we can reduce our analysis to a family of systems on $\mathbb{R}^3$. We provide the global phase portrait of these systems in the Poincaré ball (i.e., in the compactification of $\mathbb{R}^3$ with the sphere $\mathbb{S}^2$ of the infinity).
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## I. INTRODUCTION AND STATEMENT OF MAIN RESULTS

The classical equations of mathematical physics are typically linear second order differential equations. However, many problems in the sciences and in engineering are intrinsically nonlinear. The *Fisher–Kolmogorov equation*

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u - u^3$$

was originally proposed in 1937 to model the interaction of dispersal and fitness in biological populations. The *EFK-equation* or more precisely the extended Fisher–Kolmogorov equation,

$$\frac{\partial u}{\partial t} = -\gamma\frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial x^2} + u - u^3, \quad \gamma > 0$$

was proposed in 1988 as a higher order model equation for physical systems that are bistable (i.e., the EFK-equation has two uniform states $u(x) = \pm 1$ which are stable, separated by a third uniform state $u(x) = 0$). For stationary solutions, that is, the solutions which do not depend on the time $t$, the EFK-equation reduces to the ordinary differential equation

$$-\gamma\frac{d^4u}{dx^4} + \frac{d^2u}{dx^2} + u - u^3 = 0, \quad \gamma > 0.$$

By the transformation

$$x = \sqrt[4]{\gamma}\,\bar{x}, \quad q = -\frac{1}{\sqrt{\gamma}},$$

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