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GLOBAL DYNAMICS IN THE POINCARÉ BALL OF THE CHEN SYSTEM HAVING INVARIANT ALGEBRAIC SURFACES

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In this paper, we perform a global analysis of the dynamics of the Chen system

$$\dot{x} = a(y - x), \quad \dot{y} = (c - a)x - xz + cy, \quad \dot{z} = xy - bz,$$

where $(x, y, z) \in \mathbb{R}^3$ and $(a, b, c) \in \mathbb{R}^3$. We give the complete description of its dynamics on the sphere at infinity. For six sets of the parameter values, the system has invariant algebraic surfaces. In these cases, we provide the global phase portrait of the Chen system and give a complete description of the α- and ω-limit sets of its orbits in the Poincaré ball, including its boundary $\mathbb{S}^2$, i.e. in the compactification of $\mathbb{R}^3$ with the sphere $\mathbb{S}^2$ of infinity. Moreover, combining the analytical results obtained with an accurate numerical analysis, we prove the existence of a family with infinitely many heteroclinic orbits contained on invariant cylinders when the Chen system has a line of singularities and a first integral, which indicates the complicated dynamical behavior of the Chen system solutions even in the absence of chaotic dynamics.

Keywords: Chen system; integrability; Poincaré compactification; dynamics at infinity; heteroclinic orbits; singularly degenerate heteroclinic cycles; invariant manifolds.

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