Institute of Physics Publishing — Nonlinearity

Nonlinearity **19** (2006) 741–746 doi:10.1088/0951-7715/19/3/011

Negative periodic orbits for graph maps

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Received 9 September 2005, in final form 9 December 2005
Published 6 February 2006
Online at stacks.iop.org/Non/19/741

Recommended by C Liverani

Abstract

Recently, Alves, Hric and Sousa Ramos proved that for any continuous piecewise monotone map of a graph into itself, its topological entropy is equal to the maximum of two quantities. The first one is the (exponential) upper growth rate, as $n \to \infty$, of the number of periodic points of period n at which the orientation is reversed. The second one is the logarithm of the spectral radius of the map induced in the first homology group.

We extend the essential part of this theorem to arbitrary continuous maps of a graph into itself and provide a substantially shorter proof of the whole theorem.

Mathematics Subject Classification: 37E25, 37B40

1. Introduction

Recently, Alves *et al* [3] proved that for any continuous piecewise monotone map of a graph to itself, its topological entropy is equal to the maximum of two quantities. The first one is the (exponential) upper growth rate, as $n \to \infty$, of the number of periodic points of period n at which the orientation is reversed. The second one is the logarithm of the spectral radius of the map induced in the first homology group. (We will give all necessary definitions in the next section.)

This equality can be understood as follows. Since the entropy of continuous graph maps is given by horseshoes (see [5]), the upper growth rate of the number of periodic points should be equal to the entropy, if periodic points are counted in the right way (that is, sometimes we count a whole class of periodic points as one point). If there are not enough 'negative' ones, the only way we can get from a 'positive' one to the next 'positive' one is to go around some circuit in the graph, and this causes a sufficiently big spectral radius of the map in the first

