RENORMALISATION AND PERIODIC STRUCTURE FOR BIMODAL MAPS.

JAUME LLIBRE(*) AND PERE MUMBRÚ(*,**)

Abstract. For the unimodal maps the renormalisation can be interpreted in the space of kneading sequences as the operator R*. The kneading theory and the operator R* have been the most useful tools for studying the periodic structure and the behaviour of the topological entropy of the unimodal maps.

We consider continuous maps with two turning points or turning intervals and we define explicitly for this case the renormalisation operator analogous to R*. Then we show the existence of three fixed points for this operator (two sinks and one saddle) and how the periodic structure of the space of the kneading pairs is based on the self similarity and repetition of some geometric features, the stairs

1. INTRODUCTION

For the unimodal maps the $R*$ operator, defined on the space S of symbolic sequences of symbols $L's, C's$ or $R's$, shows us different properties of this space. In particular,

- $R*$ allows us to describe the relation between the kneading invariants of two different maps related by the doubling or renormalisation transformation (see [**CE**,p.199]),

- $R*$ becomes the key to show how the periods of the periodic orbits of the unimodal maps appear with respect to the lexicographical order defined in S (see [**CE**,p.78]), or

- $R*$ can be extended to a more general $*$ product, that explains the existence of "boxes within-a-box" and "boxes in files" on the sequence space S (see [**GM**] and [**Gi**]).

For other kind of maps the equivalent to the $R*$ operator is not known. For bimodal maps some authors like Mackay and Tresser [**MaT**] have shown the interest of extending the notion of $R*$ operator. We shall give this extension for the bimodal maps and we call the new operator, the mother operator.

The bimodal maps will be continuous maps with two turning points or turning intervals, like in Milnor-Thurston [**MT**] or Guckenheimer [**Gu**], respectively. The mother operator can be extended to a more general $*$ operator . Then we can prove that some structure of boxes exists for the bimodal maps, and we can show also, by using such boxes, some properties of the topological entropy. But here we present mainly the basic properties of the mother operator, and the periodic structure of the bimodal maps.