AVERAGING THEORY AT ANY ORDER FOR COMPUTING LIMIT CYCLES OF DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS WITH MANY ZONES

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Abstract This work is devoted to study the existence of periodic solutions for a family of planar discontinuous differential systems $Z(x, y; \varepsilon)$ with many zones. We show that for $|\varepsilon| \neq 0$ sufficiently small the averaged functions at any order control the existence of crossing limit cycles for systems in this family. We also provide some examples dealing with nonlinear centers when $\varepsilon = 0$.

Keywords periodic solution · averaging method · nonsmooth differential system · discontinuous differential system

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1. Introduction and statement of the main results

In the qualitative theory of real planar differential system the determination of limit cycles, defined by Poincaré [21], has become one of the main problems. The second part of the 16th Hilbert problem deals with planar polynomial vector fields and proposes to find a uniform upper bound $H(n)$ (called Hilbert's number) for the number of limit cycles that these vector fields can have depending only on the polynomial degree n. The averaging method has been used to provide lower bounds for the Hilbert number $H(n)$ see, for instance, [13]. The interest on this topic extends to what we call *discontinuous piecewise vector fields*.

The increasing interest in the theory of nonsmooth vector fields has been mainly motivated by its strong relation with Physics, Engineering, Biology, Economy, and other

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