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On the periodic solutions of a perturbed double pendulum

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To Waldyr Oliva in his 80^{th} Birthday

Abstract. We provide sufficient conditions for the existence of periodic solutions of the planar perturbed double pendulum with small oscillations having equations of motion

$$\ddot{\theta}_1 = -2a\theta_1 + a\theta_2 + \varepsilon F_1(t, \theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2),$$
$$\ddot{\theta}_2 = 2a\theta_1 - 2a\theta_2 + \varepsilon F_2(t, \theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2),$$

where a and ε are real parameters. The two masses of the unperturbed double pendulum are equal, and its two stems have the same length l. In fact $a = g/l$ where g is the acceleration of the gravity. Here the parameter ε is small and the smooth functions F_1 and F_2 define the perturbation which are periodic functions in t and in resonance p:q with some of the periodic solutions of the unperturbed double pendulum, being p and q positive integers relatively prime.

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