IOP Publishing | London Mathematical Society Nonlinearity
Nonlinearity 27 (2014) 563–583 doi:10.1088/0951-7715/27/3/563

Higher order averaging theory for finding periodic solutions via Brouwer degree

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Received 6 March 2013, revised 17 January 2014
Accepted for publication 20 January 2014
Published 25 February 2014

Recommended by A Chenciner

Abstract

In this paper we deal with nonlinear differential systems of the form

$$x'(t) = \sum_{i=0}^{k} \varepsilon^i F_i(t, x) + \varepsilon^{k+1} R(t, x, \varepsilon),$$

where $F_i : \mathbb{R} \times D \to \mathbb{R}^n$ for $i = 0, 1, \ldots, k$, and $R : \mathbb{R} \times D \times (-\varepsilon_0, \varepsilon_0) \to \mathbb{R}^n$ are continuous functions, and T-periodic in the first variable, D being an open subset of $\mathbb{R}^n$, and ε a small parameter. For such differential systems, which do not need to be of class $\mathcal{C}^1$, under convenient assumptions we extend the averaging theory for computing their periodic solutions to k-th order in ε. Some applications are also performed.

Keywords: periodic solution, averaging method, non-smooth differential system, discontinuous differential system, Brouwer degree
Mathematics Subject Classification: 34C29, 34C25, 47H11

1. Introduction and statement of the main results

The method of averaging is a classical and mature tool that allows us to study the dynamics of nonlinear *differential systems* under periodic forcing. The method of averaging has a long history that starts with the classical works of Lagrange and Laplace, who provided an intuitive justification of the method. The first formalization of this theory was done in 1928 by Fatou [11]. Important practical and theoretical contributions to the averaging theory were made in the 1930s by Bogoliubov and Krylov [2], and in 1945 by Bogoliubov [1]. In 2004,

