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**Abstract:** The study is made of the foliations of the energy levels of a class of integrable Hamiltonian systems by the sets of constant energy and angular momentum, including a classification of the topological bifurcations and a dynamical characterization of the critical leaves (separatrix surfaces) of the foliation. Then, Hamiltonian perturbations of this class of integrable Hamiltonians are considered, and conditions are given for the persistence of the separatrix structure of the foliations, and for the existence of transversal ejection-collision orbits of the perturbed system. Finally, we consider a class of non-Hamiltonian perturbations of a family of integrable systems of the type studied before, and we prove the persistence of "almost all" the tori and cylinders that foliate the energy levels of the unperturbed system as a consequence of KAM theory.

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