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MINIMUM NUMBER OF FIXED POINTS FOR MAPS OF THE FIGURE EIGHT SPACE

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Let E denote the figure eight space, and $M(f)$ the minimum number of fixed points achievable among maps homotopic to a given self-map f of E. We give an algorithm to compute $M(f)$ from the induced homomorphism f_* on the fundamental group of E.

1. Introduction

In recent years, several papers have been devoted to the question of finding the minimum number of fixed points, or the minimum periodic structure, for classes of self-maps of compact manifolds (see e.g. [Franks, 1982, 1985; Llibre, 1993a, 1993b; Casasayas *et al.*, 1994, 1995; Alsedà *et al.*, 1995; Guillamon *et al.*, 1995; Jiang & Llibre, 1996; Fagella & Llibre, 1996]), or of graphs (see e.g. [Llibre & Misiurewicz, 1993; Leseduarte & Llibre, 1996]). In particular, some of these papers address the question of finding, given a self-map f of a compact ANR X of dimension n, the minimum number $M(f)$ of fixed points occurring among maps homotopic to f.

An important homotopic invariant which arises in this context is the Nielsen number of f, $N(f)$ (see e.g. [Jiang, 1983; Kiang, 1989]), which has the property $N(f) \leq M(f)$. When X is a compact connected polyhedron without local separating points and is not a surface with negative Euler characteristic, the problem of determining $M(f)$ is solved and we have $N(f) = M(f)$ (see [Jiang, 1983] or [Nielsen, 1927, 1929, 1932; Wecken, 1941, 1942a, 1942b; Shi, 1966; Jiang, 1980] for the original references). However, there are examples when $M(f) > N(f)$. The case when X is the figure eight space and in which we may have $N(f) = 0$ and $M(f) > 0$ has been considered in several instances (see e.g. [Wecken, 1941; Llibre & Reventós, 1981]). Since the Nielsen number is a homotopy type invariant, this example extends naturally to the case when X is the closed disk with two open holes removed.

For the closed disk with two open holes removed, the problem has been solved by Kelly [1987], who gave an algorithm for the computation of $M(f)$. From this algorithm, it follows that in fact the difference between $N(f)$ and $M(f)$ can be arbitrarily large, both for maps of the closed disk with two open holes removed and for maps of the figure eight space. So, the Nielsen number gives no information about $M(f)$ for maps defined in either one of these spaces. Moreover, as we shall see in Sec. 2, given two maps, f_1 and f_2 of the closed disk with two open holes removed and of the figure eight space, respectively, such that f_1 and f_2 induce the

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