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Periodic solutions of the Nathanson's and the Comb-drive models

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ABSTRACT

We provide sufficient conditions for the existence and stability properties of periodic solutions of the second-order non-autonomous differential equation of the Nathanson's model

$$\ddot{x}+x+a\dot{x}-\frac{b(v_0+\delta v(\omega t))^2}{(1-x)^2}=0,$$

and of the Comb-drive model

$$\ddot{x}+x+a\dot{x}-\frac{4b(v_0+\delta v(\omega t))^2 x}{(1-x^2)^2}=0,$$

where $x\in\mathbb{R}$, a, b, v_0, δ are positive parameters and $v(\omega t)$ is a $2\pi/\omega$-periodic function. The results are obtained using the averaging theory and the lower and upper solution method.

1. Introduction

In Micro-Electro-Mechanical Systems (MEMS) the *electrostatic actuators* are the most used devices due to its wide variety of applications, mainly in sensing and actuation. Because the cheap production and the high performance of this technology, MEMS devices can be found all around us nowadays, for example in crash air-bag deployment systems in cars, gyroscopes for smart-phones, kidney dialysis to monitor the inlet and outlet pressures of blood (see [1]), but also in several resonant sensors [2], accelerometers [3], micro-pumps [4] and micro-valves [5]. An excellent review of the many applications of these devices can be found in [6,7] and the references there in. In the last several years a large number of electrostatic actuators have been studied both from the numerical [8–10] and experimental point of view, [10,11]. However, the number of documents devoted to a rigorously mathematical analysis of these devices is relatively low. In order to understand the dynamics of these devices, the first approach was introduced into the literature by Nathanson in [11] (1967) by means of a "lumped" mass–spring model where the elastic behavior of the system is modeled by a linear spring and the electrostatic forces are computed considering a simple parallel plate capacitor.

Up to our knowledge, the first mathematical analysis of periodic solutions for the *Nathanson's model* appears in [12] (2007) by means of shooting techniques and later in [13] (2013) using degree theory and lower and upper solution method where the authors prove the existence, multiplicity and stability of periodic solutions. Recently in [14] (2017) using the Leray–Schauder continuation theorem, the authors prove the existence of symmetric periodic solutions for other electrostatic actuator known in the literature as a *Comb-drive model*. Motivated by the successful use of well-developed mathematical techniques for the study of some canonical MEMS, the aim of this document is to provide sufficient conditions for the existence and stability properties of periodic solutions of the equation of motions of two special types of electrostatic actuators, namely, the Nathanson's model (or Parallel-Plate capacitor model) which is based on the resonant gate transistor [11] and the Comb-drive model [6,14].

Both models deals with the motion of one moveable capacitor plate under Coulomb forces and a DC–AC voltage $\hat{V}(t)=\hat{v}_0+\hat{\delta}v(\omega t)$. Here $\hat{v}_0$ represents the DC-voltage and $\hat{\delta}v(\omega t)$ represents the AC-voltage, where $v(\omega t)$ is a $2\pi/\omega$-periodic function. For the Nathanson's model the moveable plate is attached to a linear spring and moves parallel to a stationary one in a media with positive viscosity coefficient (see Fig. 1). In appropriate units, the equation motion for the moveable capacitor is given by the following non-dimensional second-order-differential equation

$$\ddot{x}+x+a\dot{x}-\frac{b\hat{V}^2(t)}{(1-x)^2}=0,$$

where $x\in(-\infty,1)$ and the dot denotes derivative with respect to the time t. For the Comb-drive model the moveable plate (*finger*) is now

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