ON THE PERIODIC SOLUTIONS OF THE MILCHELSON CONTINUOUS AND DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEM

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Abstract. Applying new results from the averaging theory for discontinuous and continuous differential systems, we study the periodic solutions of two distinct versions of the Michelson differential system: a Michelson continuous piecewise linear differential system, and a Michelson discontinuous piecewise linear differential system.

1. Introduction and statement of the main result

The Michelson differential system is given by

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= c^2 - y - \frac{x^2}{2}, \end{aligned} \tag{1}$$

with $(x, y, z) \in \mathbb{R}^3$ and the parameter $c \geq 0$. The dot denotes derivative with respect to an independent variable t, usually called the time. This system is due to Michelson [13] for studying the traveling solutions of the Kuramoto-Sivashinsky equation. It also arises in the analysis of the unfolding of the nilpotent singularity of codimension three [4, 6].

This system has been largely investigated from the dynamical point of view. In the first study of Michelson [13] he proved that if $c > 0$ is sufficiently large, then system (1) has a unique bounded solution which is a transversal heteroclinic orbit connecting the two finite singularities $(-\sqrt{2}c, 0, 0)$ and $(\sqrt{2}c, 0, 0)$. When c decreases there will appear a cocoon bifurcation (see [7, 8, 13]). A complete description of the phase portrait at infinity of system (1) via the Poincaré compactification was

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