FINAL EVOLUTIONS FOR SIMPLIFIED MULTISTRAIN/TWO-STREAM MODEL FOR TUBERCULOSIS AND DENGUE FEVER

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Abstract. The simplified multistrain/two-stream model for the tuberculosis and the Dengue fever here considered has three compartments, one susceptible and the other two infectious. We characterize all the final evolutions of this model under generic assumptions.

1. Introduction and statement of the main results

Recently it has become important the study of the resistant viral and bacterial strains, and the treatment on their proliferation, see for instance [1, 2]. A way for analyzing these systems is through the multistrain model, which **is** a simplification of the model considered by Castillo-Chavez and Feng, see section 12 of [2], for the study of the tuberculosis model of Feng-Velasco-Hernández [5] for analyzing the Dengue fever. Driessche and Watmough [4] studied this model in terms of their reprodutive numbers and subthreshold epidemic equilibrium points.

This simplified model has a unique susceptible compartment (S), but has two infectious compartiment agents (I_1, I_2). The equation of this simplified model is

$$
\begin{aligned}
\dot I_1 &= \beta_1 I_1 S - (b+\gamma_1) I_1 + \nu I_1 I_2,\\
\dot I_2 &= \beta_2 I_2 S - (b+\gamma_2) I_2 - \nu I_1 I_2,\\
\dot S &= b - bS + \gamma_1 I_1 + \gamma_2 I_2 - (\beta_1 I_1 + \beta_2 I_2) S.
\end{aligned} \tag{1}
$$

From their biological meaning the parameters of this system satisfy

$$\nu, b > 0, \quad \beta_i \geq 0 \quad \text{and} \quad b+\gamma_i > 0 \quad \text{for } i = 1,2. \tag{2}$$

System (1) has the Darboux invariant

$$I = (I_1 + I_2 + S - 1) e^{bt}.$$

See section 2 for a definition of Darboux invariant. Let Q^* be the open octant $\{I_1 > 0\} \cap \{I_2 > 0\}$ in the space $\mathbb{R}^3$ of coordinates (I_1, I_2, S). We shall see that the existence of this invariant implies that all the final evolutions of $I_1(t)$ and $I_2(t)$ when $t \to +\infty$ tend to the attractors contained in the quadrant $Q = \{I_1 + I_2 + S = 1\} \cap \{I_1 \geq 0\} \cap \{I_2 \geq 0\}$ adding the infinity, see the definition of final evolution in section 2.

We recall that an equilibrium point p of a differential system is *hyperbolic* if the real part of the eigenvalues of the linear part of that system at p are non–zero. As we shall see in section 3 the assumption that the equilibrium points of system (1) are hyperbolic **is** generic.

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